Nim is an ancient game with many variations. In each variation, players take turns removing stones from one or more piles. The variations below, in which all of the stones begin in one pile, are sometimes called "subtraction games."

5|2: The game begins with 5 stones or other objects. Players take turns removing 1 or 2 stones. The player who picks up the last stone wins. The first player can always win if he or she plays correctly. Should the first player remove 1 or 2 stones to guarantee a win?

8|3: The game begins with 8 stones, and players take turns removing 1, 2, or 3 stones. The player who picks up the last stone wins. This time, the second player can always win. Can you find a strategy that will allow the second player to win every time?

21|3: The game begins with 21 stones, and players take turns removing 1, 2, or 3 stones. The player who picks up the last stone wins. Can you discover whether it is better to play first or second? What is the winning strategy?

In each of the numbered versions above, the first number gives you the number of stones to begin with. The second number gives you the maximum number of stones a player may remove each turn. Try your own variations.

Strategy hints are on the next page.

Let's begin by exploring the game of Nim when players can remove **1 or 2 stones** each turn.

Suppose there are 3 stones on the board and it is your turn:

If you remove 2 stones, your opponent will pick up the last stone to win the game. If you remove 1 stone, your opponent will pick up the last two stones to win the game. Either way, your opponent can win. Obviously, this is a situation you want to avoid!

However, if you can *leave* 3 stones for your opponent, you can always win!

So, in a game of 5|2 Nim, you can win if you go first and remove 2 stones:

Now, your opponent must remove 1 or 2 stones. Either way, you can take the remaining stone(s) and win!

What if you start with 6 stones in a game of 6|2 Nim? Would you prefer to go 1st or 2nd?

6|2:   If the first player removes 1 stone, the second player can remove 2 stones as above and win the game.

If the first player removes 2 stones, the second player can remove 1 stone. This leaves 3 stones for the first player. So, the second player can always win again.

So, in 6|2 Nim, you can always win if you go second. In a game with more than 6 stones, if you leave 6 stones for your opponent, you can always win!

What about 7|2 Nim? Would you prefer to go 1st or 2nd?

What if you start with 8 stones? 9? 99? 100?

What do you notice about the number of stones in a game that can always be won by the first player versus the number of stones in a game that can always be won by the second player?

Let's move on to a game of Nim in which players can remove **1, 2 or 3 stones**.

Suppose there are 4 stones on the board and it is your turn:

⬤ ⬤ ⬤ ⬤

No matter how many stones you remove, your opponent will always be able to take the remaining stone(s) to win the game.

Obviously, this is a situation you want to avoid!

However, if you can *leave* 4 stones for your opponent, you can always win!
So, in a game of 5|3 Nim, you can win if you go first and remove 1 stone:

⬤ ⬤ ⬤ ⬤ ○

In a game of 6|3 Nim, you can win if you go first. How many stones should you remove?
In a game of 7|3 Nim, you can win if you go first. How many stones should you remove?

8|3: ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤ ⬤

What about a game of 8|3 Nim? If you go first, you can remove 1, 2, or 3 stones. This leaves the second player with 7, 6, or 5 stones. Do you see the problem with going first? No matter what you do, you always leave the second player in a position to win the game.

However, if you can *leave* 8 stones for your opponent, you can always win!

In a game of 9|3 Nim, would you prefer to go 1st or 2nd?
What if you start with 10 stones? 11? 12? 100?

What do you notice about the number of stones in a game that can always be won by the first player versus the number of stones in a game that can always be won by the second player?

Try lots of variations until you have mastered the Nim games above and can answer questions like this one: In a game of 31|6 Nim, would you prefer to go first or second?

Then, you are ready to try some of the variations on the next page.

Poison Nim
In the "poison" variations of Nim, the player who picks up the last stone **loses**.
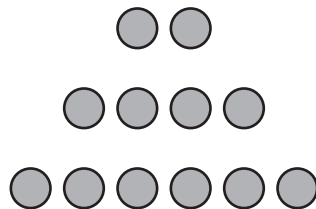In a game of 5|2 poison Nim, would you prefer to play first or second? What about 6|2? 7|2?

Multi-Row Nim (Classic Nim)
In the variations of Nim that we have discussed so far, all of the stones have been placed in one row. These games of Nim are commonly called "subtraction games."

The most common forms of Nim are played with more than one row. Players take turns removing 1 or more stones from the *same* row (a player may take all of the stones from a single row). Multi-row Nim is usually played "poison," where the person who pick up the last stone loses. Classic multi-row Nim can be played with any number of stones in any number of rows.
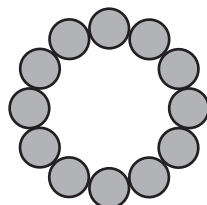
Sample Game

Player 1	Player 2	Player 1	Player 2	Player 1 is forced to pick up the last stone. So, Player 2 wins.

Circle Nim
In Circle Nim, stones are placed in a circle as shown below. Players take turns removing 1 or 2 stones. But, the stones must be next to each other (touching). Circle Nim can be played "standard" or "poison." Or, allow up to 3 stones to be removed on each turn.

Start Small!
The winning strategies to the Nim variations above are easiest to discover if you start with a small number of stones. Try classic Nim with rows of 1, 2, and 3 stones. Try circle Nim with 5 or 6 stones. Then, work your way up to more complex variations.